HEAVY QUARK FRAGMENTATION FUNCTIONS IN THE HEAVY QUARK EFFECTIVE THEORY

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Abstract

We calculate fragmentation functions for a b-quark to fragment into color-singlet P-wave bound states $\bar{c}b$ in the Heavy Quark Effective Theory with the exact account of $O(1/m_b)$ corrections. We demonstrate an agreement of the obtained results with the corresponding calculations carried out in quantum chromodynamics.

Introduction.

The study of heavy quarkonia properties is a subject of much current interest for understanding of quark-gluon interaction dynamics. B_c-mesons, consisting of b- and c-quarks, hold a unique position in the heavy quarkonia physics [1]. In the first place, B_c -mesons consist of two heavy quarks, so the predictions of potential models refer to J/Ψ- and Υ -mesons as well as to B_c -mesons [2]. In the second place, B_c -mesons are constructed from the quarks of different flavors and masses, what essentially determine their decay characteristics [3].

Production of mesons with heavy quarks in e^+e^-, $\gamma\gamma$ and $p\bar{p}$-interactions may be described in nonrelativistic perturbative quantum chromodynamics. At present, two mechanisms were investigated for the production of B_c-mesons: the recombination and the fragmentation. In the first case, B_c-mesons are formed from heavy quarks, produced independently in hard subprocess. The fragmentational mechanism demands the pair production of b- or c-quarks in hard subprocess with their subsequent fragmentation to B_c-mesons ($\bar{b} \to B_c\bar{c}, c \to B_c b$). The relative contributions of these mechanisms in the production cross-sections are different in various reactions. In e^+e^- -annihilation only quark fragmentation is essential [4]. In $p\bar{p}$ $\gamma\gamma$ -interactions the fragmentational mechanism prevails also for B_c-mesons production with large transverse momenta [5, 6]. In the range of small transverse momenta the recombination is dominant and determines the total cross section of B_c-meson production in $\gamma\gamma$ and $p\bar{p}$ interactions. But experimental conditions of B_c-meson discovery are more perspective in the large transverse momenta domain. So, the study of heavy quark fragmentation into B_c-mesons attracts considerable interest. An approach for the calculation of fragmentation functions $D_{\bar{b}\to B_c\bar{c}}, D_{c\to B_c b}$ in nonrelativistic perturbative quantum chromodynamics was suggested in [7].

At the same time, in the last years there was suggested, based on QCD, the Heavy Quark Effective Theory (HQET) [8, 9] for description of heavy hadrons properties. HQET make it possible to obtain the finite analytical result with some accuracy even for complicated processes of quark- gluon interaction. In this approach the matrix elements of

different processes may be decomposed on degrees of two small parameters: the strong coupling constant $\alpha_s(m_Q)$ and Λ_{QCD}/m_Q, where m_Q is the mass of the heavy quark. In the limit $m_Q \to \infty$ the effective lagrangian, which describes the strong interactions of heavy quarks has an exact spin-flavor symmetry [8]. HQET is successfully used for investigation of exclusive and inclusive hadron decays [9]. Recently it was shown [10] that HQET may be used for study of b-quark fragmentation into S-wave pseudoscalar and vector mesons and the corresponding nonpolarized fragmentation functions were calculated. The HQET calculation of the b-quark fragmentation into the transverse and longitudinal polarized S-wave B_c^*-mesons have been made in [11]. In this work we have calculated the fragmentation functions of b-quark into P-wave color-singlet states $(\bar{c}b)$ to the next to leading order in the heavy quark mass expansion using the methods of HQET.

1 Fragmentation functions into P-wave $\bar{c}b$-mesons.

Heavy b-quark may fragment into bound states of two heavy quarks ($\bar{c}b$ - states) with orbital momentum l=1. There are four such states: ${}^1P_1, {}^3P_J$ (J=0,1,2). Heavy quark fragmentation functions into P-wave B_c- mesons were calculated by Chen [12] and Yuan [13] in QCD, but the results of their calculations disagree. Let carry out the similar calculation of fragmentation functions in the HQET. Let $q = m_b v + k$ is 4-momentum of virtual heavy quark, $p_1 = (1-r)Mv + \rho$ and $p_2 = rMv - \rho$ are 4-momenta of $b-$ and $\bar{c}$-quarks correspondingly; ρ is 4-momentum of relative motion. Let also $l = k - \rho$ is 4-momentum of the virtual gluon and k is the residual momentum of the fragmenting heavy quark. Fragmentation functions for the process $b \to B_c + c$ are determined by the next expression [7]:

$$D(z) = \frac{1}{16\pi^2} \int ds \theta \left(s - \frac{M^2}{z} - \frac{m_c^2}{1-z} \right) \lim_{q_0 \to \infty} \frac{\sum |T|^2}{\sum |T_0|^2}, \tag{1}$$

where $M = m_b + m_c$ is the mass of B_c-meson, T is the matrix element for production $B_c + \bar{c}$ from an off-shell b^*-quark with virtuality $s = q^2$, and T_0 is the matrix element for producing an on-shell b-quark with the same 3-momentum $\vec{q}$. The calculation can be greatly simplified by using the axial gauge with gauge parameter $n_\mu = (1, 0, 0, -1)$ in the frame where $q_\mu = (q_0, 0, 0, \sqrt{q_0^2 - s})$:

$$D_{\sigma\lambda}(k) = \frac{1}{k^2 + i0} \left[g_{\sigma\lambda} - \frac{k_\sigma n_\lambda + k_\lambda n_\sigma}{k \cdot n} + \frac{n^2 k_\sigma k_\lambda}{(k \cdot n)^2} \right], \tag{2}$$

The part of amplitude T that involves production of the virtual b^*-quark can be treated as an unknown Dirac spinor Γ. In the limit $q_0 \to \infty$, the same spinor factor Γ appears in the matrix element $T_0 = \bar{\Gamma} v(q)$, what leads to cancellation of this factor Γ in (1).

Let consider the fragmentation of b-quark into color-singlet bound state $(\bar{c}b)$ ${}^1P_1$. The amplitude of such process involves the spinor factor $v(p_1)\bar{u}(p_2)$. To project the pair of quarks on ${}^1P_1$ bound state we have used the next substitution [14]:

$$v(p_2)\bar{u}(p_1) \to \sqrt{M} \frac{\hat{p}_2 - m_c}{2m_c} \gamma_5 \frac{\hat{p}_1 + m_b}{2m_b}. \tag{3}$$

The HQET Lagrangian, including the leading and the $1/m_b$ terms is given by [8, 9]:

$$L = \bar{h}_v \left\{ iv \cdot D + \frac{1}{2m_b} \left[C_1 (iD)^2 - C_2 (v \cdot iD)^2 - \frac{C_3}{2} g_s \sigma_{\mu\nu} G^{\mu\nu} \right] \right\} h_v, \tag{4}$$

where

$$C_1 = 1, \quad C_2 = 3 \left(\frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right)^{-\frac{8}{(33-2n_f)}} - 2, \quad C_3 = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right)^{-\frac{9}{(33-2n_f)}}. \tag{5}$$

All of these coefficients are equal to 1 at the heavy quark mass scale $\mu = m_b$. When we concerned the fragmentation into P-wave mesons, it is necessary to decompose the projecting operator (3) and the gluon propagator (2) on the relative motion momentum ρ. Using the Feynman rules, derived from HQET Lagrangian (4), we may write the full fragmentation amplitude into $^1P_1$ -state [14]:

$$iM(n^1P_1) = \frac{\sqrt{4\pi M}\alpha_s}{3m_c m_b} R_1'(0) \epsilon_\alpha(L_z) \frac{\partial}{\partial \rho_\alpha} \Big\{ \frac{1}{l^2} (-g_{\mu\nu} + \frac{n_\nu l_\mu}{n \cdot l}) \tag{6}$$

$$\Big\{ \bar{u}(p') \gamma^\nu (m_c \hat{v} - \hat{\rho} - m_c) \gamma_5 (m_b \hat{v} + \hat{\rho} + m_b) [v^\mu + \frac{C_1}{2m_b} (\rho + k)^\mu -$$

$$- \frac{C_2}{2m_b} v \cdot (\rho + k) v^\mu + i \frac{C_3}{2m_b} \sigma^{\mu\lambda} (\rho - k)_\lambda] \frac{1 + \hat{v}}{2} \Gamma \frac{i}{v \cdot k + \frac{C_1}{2m_b} k^2 - \frac{C_2}{2m_b} (v \cdot k)^2} \Big\} |_{\rho = 0},$$

where $\epsilon_\alpha(L_z)$ is the polarization vector of $^1P_1$ -state. To calculate amplitude (6) it is convenient to divide it into two parts on the degrees of small parameter $1/m_b$. When $m_b \to \infty$ in vertex function and in the propagator of heavy quark, we obtain the main contribution to the fragmentation amplitude of b-quark in the form:

$$iM_1(n^1P_1) = \frac{\sqrt{4\pi M}\alpha_s 2R_1'(0)}{3r^2(s - m_b^2)^3} \epsilon^*_\alpha(L_z) \bar{u}(p') W^\alpha \gamma_5 \Gamma, \tag{7}$$

$$W_\alpha = (s - m_b^2) \left[(\hat{v} + 1)\gamma_\alpha - \frac{v \cdot k}{n \cdot k} \hat{n} (\hat{v} + 1) \gamma_\alpha \right] + 4mk_\alpha \left[1 + \frac{k \cdot v}{k \cdot n} \hat{n} \right] (\hat{v} - 1) -$$

$$-2Mr(s - m_b^2) v_\alpha \frac{1}{n \cdot k} \hat{n} (\hat{v} - 1) + 2Mr(s - m_b^2) n_\alpha \frac{v \cdot k}{(n \cdot k)^2} \hat{n} (\hat{v} - 1).$$

All calculations of the fragmentation functions, which are rather complicated, were done by means of the system "REDUCE". Substituting (7) into (1), we obtain:

$$D_1(n^1P_1)(y) = N_1 \frac{(1-y)^2}{ry^8} (9y^4 - 4y^3 + 40y^2 + 96), \quad N_1 = \frac{\alpha_s^2 |R'_{nP}(0)|^2}{54\pi r^5 M^5}. \tag{8}$$

where $y = (1 - z + rz)/rz$ is the so called Yaffe-Randall variable [15], and $r = m_c/M$.

The amplitude of bound $\bar{c}b$ state n^3P_J production may be derived from (6), changing $\gamma_5 \to \hat{\epsilon}(S_z)$, where $\epsilon^\mu(S_z)$ is the spin wave function:

$$iM(n^3P_J) = \frac{\sqrt{4\pi M}\alpha_s}{3m_c m_b} R_1'(0) \sum_{S_z,L_z} < 1, L_z; 1, S_z | J, J_z > \epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z) \tag{9}$$

$$\frac{\partial}{\partial \rho_\alpha}\Big\{\frac{1}{l^2}(-g_{\mu\nu} + \frac{n_\nu l_\mu}{n\cdot l})\bar{u}(p')\gamma^\nu(m_c\hat{v} - \hat{\rho} - m_c)\gamma_\beta(m_b\hat{v} + \hat{\rho} + m_b)[v^\mu + \frac{C_1}{2m_b}(\rho+k)^\mu -$$
$$-\frac{C_2}{2m_b}v\cdot(\rho+k)v^\mu + i\frac{C_3}{2m_b}\sigma^{\mu\lambda}(\rho-k)_\lambda]\frac{1+\hat{v}}{2}\Gamma\frac{i}{v\cdot k + \frac{C_1}{2m_b}k^2 - \frac{C_2}{2m_b}(v\cdot k)^2}\Big\}|_{\rho=0},$$

where we have expressed the Clebsch-Gordon coefficients and $\epsilon^*_\beta(S_z), \epsilon^*_\alpha(L_z)$ by the bound state polarizations [14]:

$$\sum_{S_z,L_z} < 1, L_z; 1, S_z | J, J_z > \epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z) = \left\{ \begin{array}{ll} \frac{1}{\sqrt{3}}(g_{\alpha\beta} - v_\alpha v_\beta), & J=0 \\ \frac{i}{\sqrt{2}}\epsilon_{\alpha\beta\lambda\rho}v_\lambda\epsilon^*_\rho(J_z), & J=1 \\ \epsilon_{\alpha\beta}(J_z), & J=2. \end{array}\right. \tag{10}$$

Taking in (9) only the terms of leading order on $1/m_b$ and doing necessary differentiation on ρ_α, we have obtained:

$$iM_1(n^3P_J) = \frac{\sqrt{4\pi M}\alpha_s 2R_1'(0)}{3r^2(s-m_b^2)^3} \sum_{S_z,L_z} < 1, L_z; 1, S_z | J, J_z > \epsilon^*_\beta(S_z)\epsilon^*_\alpha(L_z) \tag{11}$$

$$\bar{u}(p')\Big\{(s-m_b^2)(1-\frac{k\cdot v}{k\cdot n}\hat{n})\gamma_\alpha\gamma_\beta - 4k_\alpha M(1+\frac{k\cdot v}{k\cdot n}\hat{n})\gamma_\beta - 4rM^2\left(\frac{k\cdot v}{k\cdot n}\right)^2 n_\alpha\hat{n}\gamma_\beta\Big\}(1+\hat{v})\Gamma.$$

This amplitude determines the basic contribution to fragmentation functions of b-quark into ^3P_J- state:

$$D_1(n^3P_0)(y) = N_1\frac{(y-1)^2}{ry^8}(y^4 - 4y^3 + 8y^2 + 32), \tag{12}$$

$$D_1(n^3P_1)(y) = N_1\frac{2(y-1)^2}{ry^8}(3y^4 - 4y^3 + 16y^2 + 48), \tag{13}$$

$$D_1(n^3P_2)(y) = N_1\frac{20(y-1)^2}{ry^8}(y^4 + 4y^2 + 8). \tag{14}$$

The heavy quark fragmentation functions into P-wave mesons were calculated in [13] using the QCD. Our results (8), (12)-(14) coincide with the calculations of [13] in the leading order on $1/m_b$. Let consider calculation of the next-to-leading order contributions $O(1/m_b)$ for $^1P_1$ -state. First of all, let observe, that the fragmentation functions, determined by the amplitudes (9) and (11), involves factor $1/[1 + r(y-1)]$. Decompose it on degrees of r, we have the correction to (8):

$$D_2(n^1P_1)(y) = N_1\frac{(1-y)^3}{y^8}(9y^4 - 4y^3 + 40y^2 + 96), \tag{15}$$

Without any calculations we can obtain the propagator correction $O(1/m_b)$ to (8), which arises as a result of following substitution:

$$\frac{1}{v\cdot k}\to\frac{1}{v\cdot k+\frac{C_1}{2m_b}k^2-\frac{C_2}{2m_b}(v\cdot k)^2}\approx\frac{1}{v\cdot k}\left[1+r(-C_1+\frac{1}{2}C_2My)\right]. \tag{16}$$

This correction has the next form:

$$D_3(n^1P_1)(y)=N_1\frac{(y-1)^2}{y^8}(9y^4-4y^3+40y^2+96)(-2C_1+C_2My). \tag{17}$$

The vertex correction of necessary order in (6) is calculated in a more tedious way:

$$iM_{vert.}(n^1P_1)=\frac{\sqrt{4\pi M}\alpha_s}{3m_cm_b}R'_{nP}(0)\epsilon^*_\alpha(L_z)\frac{\partial}{\partial\rho_\alpha}\Big\{\bar{u}(p')\gamma_\nu(m_c\hat{v}-\hat{\rho}-m_c)\gamma_5 \tag{18}$$

$$(m_b\hat{v}+\hat{\rho}+m_b)\left[\frac{C_1}{2m_b}(\rho+k)_\mu-\frac{C_2}{2m_b}v\cdot(\rho+k)v_\mu+i\frac{C_3}{2m_b}\sigma_{\mu\lambda}(k-\rho)_\lambda\right]$$

$$\frac{1+\hat{v}}{2}\Gamma\frac{i}{v\cdot k}\frac{1}{l^2}\left(-g_{\mu\nu}+\frac{n_\nu l_\mu}{n\cdot l}\right)\Big\}|_{\rho=0}.$$

It is natural to perform it as a sum of several terms. Putting $\rho=0$ in the square brackets, we obtain:

$$\left[\frac{C_1}{2m_b}(\rho+k)_\mu-\frac{C_2}{2m_b}v\cdot(\rho+k)v_\mu-\frac{C_3}{4m_b}[\gamma_\mu(\hat{\rho}-\hat{k})-(\hat{\rho}-\hat{k})\gamma_\mu]\right]|_{\rho=0}= \tag{19}$$

$$=\left[\frac{C_1}{2m_b}k_\mu-\frac{C_2}{2m_b}(v\cdot k)v_\mu+\frac{C_3}{4m_b}(\gamma_\mu\hat{k}-\hat{k}\gamma_\mu)\right].$$

An addendum $(-C_2(k\cdot v)v_\mu/2m_b)$ in (19) gives the contribution to fragmentation functions which differs only by sign from the similar quark propagator correction:

$$D_4(n^1P_1)(y)=N_1\frac{(y-1)^2}{y^8}(9y^4-4y^3+40y^2+96)(-C_2My). \tag{20}$$

Two other terms in square brackets of (19) give rise the next fragmentation amplitudes:

$$iM^{(1)}_{vert.}(n^1P_1)=\frac{\sqrt{4\pi M}2\alpha_s}{3m_cm_b}R'_{nP}(0)\epsilon^*_\alpha(L_z)\frac{C_1}{2m_b}\frac{1}{r(s-m_b^2)^2}\bar{u}(p')\Big[2k_\alpha Mr\hat{n}\frac{(s-m_b^2)}{k\cdot n}- \tag{21}$$

$$-4k_\alpha m\hat{k}+2n_\alpha\hat{n}Mr^2\frac{(s-m_b)^2}{(k\cdot n)^2}+r\hat{n}\gamma_\alpha\frac{(s-m_b^2)^2}{k\cdot n}-\hat{k}\gamma_\alpha(s-m_b^2)\Big]\gamma_5(1+\hat{v})\Gamma,$$

$$iM^{(2)}_{vert.}(n^1P_1)=\frac{\sqrt{4\pi M}2\alpha_s}{3m_cm_b}R'_{nP}(0)\epsilon^*_\alpha(L_z)\frac{C_3}{m_b}\frac{1}{r^2(s-m_b^2)^2} \tag{22}$$

$$\bar{u}(p')\Big\{-2k_\alpha Mr\hat{n}\frac{(s-m_b^2)}{k\cdot n}-8k_\alpha M\hat{k}-2(s-m_b^2)k_\alpha+2Mr\frac{(s-m_b^2)}{k\cdot n}\hat{n}\gamma_\alpha\hat{k}+$$
$$+r\hat{n}\gamma_\alpha\frac{(s-m_b^2)^2}{k\cdot n}+r(s-m_b^2)\hat{k}\gamma_\alpha\Big\}\gamma_5(1+\hat{v})\Gamma.$$

A further part of vertex correction $O(1/m_b)$ appears, when we differentiate the expression in square brackets of (18) on ρ_α:

$$iM^{(3)}_{vert.}(n^1P_1)=\frac{\sqrt{4\pi M}\alpha_s}{3m_cm_b}R'_{nP}(0)\epsilon^*_\alpha(L_z)\frac{1}{2r(s-m_b^2)^2} \tag{23}$$
$$\bar{u}(p')\Big[C_1\left(k_\alpha\hat{n}\frac{1}{k\cdot n}-\gamma_\alpha\right)+C_3\Big(k_\alpha\hat{n}\frac{1}{k\cdot n}+\frac{1}{2M}\frac{(s-m_b^2)}{k\cdot n}\hat{n}\gamma_\alpha-$$
$$\frac{1}{k\cdot n}\hat{n}\hat{k}\gamma_\alpha+2\gamma_\alpha\Big)\Big]\gamma_5(1+\hat{v})\Gamma.$$

The contributions of matrix elements (21)-(23) to fragmentation functions have the following form:

$$D_5(n^1P_1)(y)=4N_1\frac{(y-1)^2y}{y^8}(3y^3-2y^2+6y+32)C_1, \tag{24}$$

$$D_6(n^1P_1)(y)=4N_1\frac{(y-1)y^2}{y^8}(-3y^3-4y^2+14y-16)C_3, \tag{25}$$

$$D_7(n^1P_1)(y)=2N_1\frac{(y-1)^2y}{y^8}\left[(3y^3+6y^2+4y+32)C_1+2y(3y^2-2y-16)C_3\right]. \tag{26}$$

The small component of heavy quark field also leads to the correction of type O(r) in function D(y). Substituting the small quark component propagator to (6) instead of heavy quark propagator

$$\frac{i}{v\cdot k}\frac{1+\hat{v}}{2}\left[\frac{1}{2m_b}\sigma_{\mu\lambda}k^\lambda\right]\frac{1-\hat{v}}{2}, \tag{27}$$

we obtain the next amplitude of B_c- meson production:

$$iM_{prop.}(n^1P_1)=-\frac{\sqrt{4\pi M}2\alpha_s}{3m_cm_b}R'_{nP}(0)\epsilon^*_\alpha(L_z)\frac{4}{r^2(s-m_b^2)^3} \tag{28}$$
$$\bar{u}(p')\left[4k_\alpha M\hat{k}+2k_\alpha(s-m_b^2)+r\frac{(s-m_b^2)^2}{k\cdot n}\hat{n}\gamma_\alpha+(s-m_b^2)\hat{k}\gamma_\alpha\right]\gamma_5(1-\hat{v})\Gamma.$$

This amplitude gives the contribution to fragmentation function of the kind:

$$D_8(n^1P_1)(y)=6N_1\frac{(y-1)y}{y^8}(y^5+2y^4+3y^3+16y-16). \tag{29}$$

The calculation of b-quark fragmentation functions into ${}^3P_J$ states was done in a similar way. The results of $D(n^3P_J)$ calculations are represented in the next section.

2 Discussion of the results.

Let analyse b-quark fragmentation functions into P-wave mesons. As was mentioned earlier, in the leading order on $1/m_b$ the expressions (8), (12)- (14) coincide with QCD calculations of Yuan [13]. The correction $O(r)$ to (8) is determined by the sum of terms (15), (17), (20), (24)-(26), (29). Setting $C_1 = C_3 = 1$, we obtain the fragmentation function $D(n^1P_1)$ to the next-to-leading order on r:

$$D(n^1P_1)(y) = \frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5 M^5}\frac{(y-1)^2}{ry^8}\Big[(9y^4-4y^3+40y^2+96)- \tag{30}$$

$$-r(3y^5-31y^4+32y^3+8y^2-192y+96)\Big].$$

Similarly, for the ${}^3P_J$ states, we have:

$$D({}^3P_0)(y) = \frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5 M^5}\frac{(y-1)^2}{ry^8}\Big[(y^4-4y^3+8y^2+32)+ \tag{31}$$

$$+\frac{r}{3}(3y^5-11y^4+392y^2+192y-96)\Big],$$

$$D({}^3P_1)(y) = \frac{\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5 M^5}\frac{(y-1)^2}{ry^8}\Big[(3y^4-4y^3+16y^2+48)+ \tag{32}$$

$$+r(3y^5+5y^4+8y^3+32y^2+96y-48)\Big],$$

$$D({}^3P_2)(y) = \frac{10\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5 M^5}\frac{(y-1)^2}{ry^8}\Big[(y^4+4y^2+8)+ \tag{33}$$

$$+\frac{r}{15}(-3y^5-y^4+36y^3-164y^2+240y-120)\Big].$$

It follows immediately from (30)-(33), that our results coincide with the calculations of Yuan [13] with the accuracy $O(r)$, if we take into account, that the nonperturbative factor of (30)-(33) in [13] contains reduced mass μ contrary to our factor rM. Using obtained fragmentation functions (30)-(33), we may calculate the fragmentation probabilities of corresponding $(\bar{c}b)$-mesons [7]:

$$P_{b\to\bar{c}b}(n^1P_1) = \int_0^1 dz D_{b\to\bar{c}b}(n^1P_1)(z,r) = \tag{34}$$

$$= \frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5 M^5}\Big[\frac{r\ln r}{(1-r)^8}\left(21+32r+110r^2-184r^3-387r^4-168r^5\right)+$$

$$+\frac{1}{210(1-r)^7}\left(1032+4497r+21353r^2-2762r^3-65202r^4-76199r^5-3679r^6\right)\Big].$$

$$P_{b\to\bar{c}b}(n^3P_0) = \frac{\alpha_s^2|R'_{nP}(0)|^2}{54\pi r^5 M^5}\Big[\frac{r\ln r}{3(1-r)^8}\left(3+8r+98r^2+1496r^3-1221r^4-960r^5\right)+ \tag{35}$$

$$+\frac{1}{630(1-r)^7}\left(360+1767r-14977r^2+180778r^3+158658r^4-427697r^5-19849r^6\right)\Big].$$

$$P_{b\to\bar{c}b}(n^3P_1)=\frac{\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5M^5}\Big[\frac{r\ln r}{(1-r)^8}\left(3+16r+10r^2+112r^3-237r^4-192r^5\right)+ \tag{36}$$

$$+\frac{1}{210(1-r)^7}\left(348-177r+8419r^2+2714r^3+14334r^4-81769r^5-4349r^6\right)\Big].$$

$$P_{b\to\bar{c}b}(n^3P_2)=\frac{10\alpha_s^2|R'_{nP}(0)|^2}{27\pi r^5M^5}\Big[\frac{r\ln r}{15(1-r)^8}\left(33-68r+130r^2-548r^3-291r^4+24r^5\right)+$$

$$+\frac{1}{3150(1-r)^7}\left(1710+2697r-2525r^2-10330r^3-146760r^4+3425r^5+583r^6\right)\Big]. \tag{37}$$

Putting here $|R'_{2P}(0)|^2=0.201$ GeV^5, $m_c=1.5$ GeV, $m_b=4.9$ GeV and $\alpha_s(2m_c)$=0.38 ($2m_c$ is a minimal energy of exchanged gluon), we have obtained the numerical value of the fragmentation probabilities, which are presented in table. We see, that our integral probabilities of P-wave $\bar{c}b$ meson production, founded by means of the b-quark fragmentation functions in the HQET are in good agreement with the results of QCD calculations of [13].

$P_{b\to\bar{c}b}(2^1P_1)$	$P_{b\to\bar{c}b}(2^3P_0)$	$P_{b\to\bar{c}b}(2^3P_1)$	$P_{b\to\bar{c}b}(2^3P_2)$
$6.4\cdot10^{-5}$	$2.5\cdot10^{-5}$	$7.3\cdot10^{-5}$	$10.5\cdot10^{-5}$

So the performed calculations show that the Heavy Quark Effective Theory may be successfully used for the study of the heavy quark fragmentation. In this approach we may systematically take into account the $O(1/m_b)$ corrections in the amplitudes and the probabilities of the fragmentation, what increases the accuracy of HQET calculations. Moreover, it seems more important, that HQET leads to finite analytical answer, when we study complicated problems in the heavy quark physics. The approach, based on HQET, may be used for calculation of heavy quark fragmentation functions into D-wave mesons, and for the investigation of B_c -meson hadroproduction.

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